Politecnico di Torino

Department of Mathematical Sciences (DISMA) “G.L. Lagrange”

Department of Excellence 2018-2022

Trails in Quantum Mechanics and Surroundings 2018

Programme & Abstracts

September 27-29, 2018

PROGRAMME

Thursday 27th:

8:45-9:00	–	Registration.
9:00-9:45	–	**GRIESEMER Marcel**: *The 2d Fermi polaron as a limit of Schrödinger operators.*
9:50-10:35	–	**BARLETTI Luigi**: *A model of diffusive transport in graphene across a quantum interface.*
10:35-11:10	–	Coffee break.
11:10-11:55	–	**BOCCATO Chiara**: *Bogoliubov theory in the Gross-Pitaevskii limit.*
12:00-12:45	–	**FALCONI Marco**: *Derivation of Ionization Models from Particle-Field Microscopic Interactions.*
12:45-14:30	–	Lunch
14:30-15:15	–	**PIZZICHILLO Fabio**: *The Hardy inequality and the Dirac-Coulomb operator.*
15:20-16:05	–	**TILLI Paolo**: *NLS ground states on metric trees.*
16:05-16:40	–	Coffe break.
16:40-17:25	–	**BANICA Valeria**: *Evolution of polygonal lines by the binormal flow.*
17:30-18:15	–	**SEIFERT Cristian**: *Spectral Monotonicity for Schrödinger Operators on Metric Graphs.*

Friday 28th:

9:00-9:45	–	**MORETTI Valter**: *Why does (relativistic) quantum theory needs complex Hilbert spaces?*
9:50-10:35	–	**MAZZUCCHI Sonia**: *Infinite dimensional integration techniques and applications to quantum mechanics.*
10:35-11:10	–	Coffee break.
11:10-11:55	–	**SAMBRI Alessia**: *Controlling the strain relaxation mechanisms in epitaxial thin films heterostructures.*
12:00-12:45	–	**MORINI Massimiliano**: *A variational approach to epitaxially strained elastic films.*
12:45-14:30	–	Lunch
14:30-15:15	–	**TROMBETTONI Andrea**: *Junctions of weakly-coupled ultracold systems.*
15:20-16:05	–	**FINCO Domenico**: *The Nobility of zero-energy Resonances.*
16:05-16:40	–	Coffe break.
16:40-17:25	–	**Round Table.**
17:30-18:15	–	**FIGARI Rodolfo**: *TBA.*
20:00	–	Social Dinner.

Saturday 29th:

9:00-9:45	–	**VISCIGLIA Nicola**: *Invariant and quasi-invariant Gaussian measures associated with dispersive PDEs.*
9:50-10:35	–	**CACCIAFESTA Federico**: *A Dirac field interacting with point nuclear dynamics.*
10:35-11:10	–	Coffee break.
11:10-11:55	–	**LAMPART Jonas**: *Boundary conditions in polaron and Nelson models.*
12:00-12:45	–	**BORRELLI William**: *Nonlinear Dirac equations on graphs with localized nonlinearities: bound states and non relativistic limit.*
12:50-13:35	–	**GRILLO Alfio**: *Thermal behaviour of the energy spectrum of "self-trapped" bosons in 1D harmonic lattices.*
13:35	–	Lunch Buffet.

ABSTRACTS

Evolution of polygonal lines by the binormal flow

Valeria BANICA
(Université Pierre et Marie Curie)

The binormal flow is a 3-D curve evolution equation that models the dynamics of filament vortices in 3-D fluids. The first part of this talk will be dedicated to the presentation of the classical connection established between the binormal flow and the 1-D cubic Schrödinger equation. We shall see in particular that the 1-D cubic Schrödinger equation with initial data a Dirac mass is related to the formation in finite time of a corner by the binormal flow. As a first result we construct solutions of the 1-D cubic Schrödinger equation in link with initial data a sum of several Dirac masses. Then we shall construct and describe a new class of singular solutions of the binormal flow, that in particular we continue after the time of singularity formation. This is a joint work with Luis Vega.

A model of diffusive transport in graphene across a quantum interface

Luigi BARLETTI
(Università degli Studi di Firenze)

We present a drift-diffusion model for stationary electron transport in graphene, in presence of sharp potential profiles such as barriers and steps.

Assuming the electric potential to have steep variations within a strip of vanishing width on a macroscopic scale, such strip is viewed as a quantum interface that couples two "classical" regions, where electron and hole transport is described in terms of semiclassical kinetic equations. The two classical regions are coupled by quantum transmission conditions, depending on the scattering coefficients.

The diffusive limit of such kinetic model is derived by means of a Hilbert expansion and a boundary layer analysis, and consists of drift-diffusion equations coupled by quantum diffusive transmission conditions across the interface. We show that, remarkably, phenomena peculiar to ballistic quantum transport, such as Klein tunnelling and angular resonances, partially survive the diffusive limit.

Bogoliubov theory in the Gross-Pitaevskii limit

Chiara BOCCATO
(Institute of Science and Technology Austria)

We consider systems of N bosons trapped in a box with volume one and interacting through a repulsive potential with scattering length of the order $1/N$ (Gross-Pitaevskii regime). For sufficiently weak interactions, we determine the low-energy spectrum up to errors that vanish in the limit of large N. Our result confirms Bogoliubov's prediction. This talk is based on joint work with C. Brennecke, S. Cenatiempo and B. Schlein.

Nonlinear Dirac equations on graphs with localized nonlinearities: bound states and non relativistic limit

William BORRELLI
(Université Paris-Dauphine)

I will present some recent results on the nonlinear Dirac (NLD) equation on metric graphs with localized nonlinearities, in the case of Kirchhoff-type vertex conditions. Precisely, I will discuss existence and multiplicity of the bound states (arising as critical points of the NLD action functional) and I will show that, in the L^2-subcritical case, they converge to the bound states of the NLS equation in the non relativistic limit. This is a joint work with R. Carlone and L. Tentarelli.

A Dirac field interacting with point nuclear dynamics

Federico CACCIAFESTA
(Università degli Studi di Padova)

In this talk we study the well posedness for a system describing a single Dirac electron coupled with classically moving point nuclei. We prove local existence of solutions for data in H^s with $s > 1$ and local well posedness in H^s for $s > 3/2$. As a main ingredient, and in fact as a result of independent interest, we construct a two-parameter propagator for the Dirac operator with several moving Coulomb singularities.

Derivation of Ionization Models from Particle-Field Microscopic Interactions

Marco FALCONI
(Eberhard Karls Universität Tübingen)

In this talk I will discuss how ionization dynamical models given by a quantum particle evolving under the action of a time-dependent point-like interaction can be derived from microscopic particle-field coupled models, in the quasi-classical limit. In particular, I will focus on the microscopic model consisting of a three-dimensional quantum particle in interaction with a phonon field, with the latter in a coherent configuration that becomes singular in the limit. It is proved that the partial trace (w.r.t. field's degrees of freedom) of particle's observables, evolved in time by the microscopic evolution, converge weakly in the quasi-classical limit to the same observables evolved by the ionization dynamics. Based on a joint work with Raffaele Carlone, Michele Correggi, and Marco Olivieri.

Title: TBA

Rodolfo FIGARI
(Università degli Studi di Napoli Federico II)

TBA

The Nobility of zero-energy Resonances

Domenico FINCO
(Università telematica internazionale UniNettuno)

We report on some recent results on Feshbach resonances. Joint work with A.Teta, R.Carlone, M.Correggi.

The 2d Fermi polaron as a limit of Schrödinger operators

Marcel GRIESEMER
(Universität Stuttgart)

In this talk I will report on new results concerning the approximation of the Fermi polaron in terms of scaled Schrödinger operators. The proofs are based on ideas developed in previous work, where we defined the Fermi polaron as a resolvent limit of ultraviolett regularized second quantized Hamiltonians. - This is joint work with Ulrich Linden and Michael Hofacker.

Thermal behaviour of the energy spectrum of "self-trapped" bosons in 1D harmonic lattices[1]

Alfio GRILLO
(Politecnico di Torino)

This contribution addresses some aspects of the role of temperature on the interactions between the harmonic oscillations of a one-dimensional lattice and a system of bosonic quasi-particles [1,2]. The lattice represents a molecular chain, whereas the bosons may be identified with the excitations of the chain's molecules generated by the release of external energy.

The interactions, modelled on the basis of a Fröhlich-type Hamiltonian [3], and studied by employing a mean-field method within the zero-order adiabatic approximation, are shown to give rise to "self-trapped" states for the bosons.

The scope of the study is to determine the energy spectrum of the "self-trapped" bosons, and to study its behaviour in response to temperature variations within an thermal range adequate for biological problems [4]. The motivation for accomplishing this task is that the self-trapping of the bosons is accompanied by the onset of Davydov-like solitons [5], which are able to answer several biological questions, such as the transport of energy in biological structures.

For the purposes of the work, the density matrix formalism is adopted, and a Helmholtz free energy is introduced and minimised in order to determine the thermodynamic ground state of the system. In conjunction with this calculation, the energy spectrum of the bosons is obtained within the long-wave approximation and the effective mass approximation as the solution of a stationary non-linear Schrödinger equation, and is proven to exhibit a bound state and a gap, whose width is related to temperature. It is found that there exists a critical number of bosons in the bound state, which may be viewed as the "minimal number of bosons" required for triggering collective phenomena, as is the case for solitons [2].

References

[1] A.A. Eremko. Phys. Rev. B 50, 5160 (1994); Phys. Rev. B 46, 3721 (1992).
[2] L.S. Brizhik, A.A. Eremko, G. Giaquinta, A. Grillo. *Preprint* (2005).
[3] H. Fröhlich. Proc. R. Soc. London Ser. A 223, 296 (1954).
[4] A.S. Davydov, *Solitons in Molecular Systems* (Dordercht: Reidel, 1985).
[5] A.S. Davydov, *Biology and Quantum Mechanics* (Pergamon Press Ltd., 1982).

Boundary conditions in polaron and Nelson models

Jonas LAMPART
(Laboratoire Interdisciplinaire Carnot de Bourgogne)

I will present a novel approach to quantum mechanical models in which bosons can be created and annihilated by nonrelativistic "source"-particles, which uses generalised boundary conditions. Hamiltonians for such models are often not given by relatively (operator-) bounded perturbations of a noninteracting Hamiltonian. They are thus usually defined using quadratic forms or renormalisation. In general, these techniques do not give much information on the domain of self-adjointness.

A recent result (arXiv:1803.00872, in collaboration with J. Schmidt) gives a definition of such Hamiltonians and their domains in terms of generalised boundary conditions. These conditions

[1]The title of this contribution rephrases the title of a research conducted by Larissa S. Brizhik, Alexander A. Eremko, Gaetano Giaquinta and myself in 2004 and 2005 [2], on which the content of this abstract is based.

are similar to those appearing in the theory of point interactions and relate sectors with different boson numbers. Our result applies, in particular, to the Fröhlich polaron and the Nelson model. I will discuss the relation of this definition to renormalisation methods. I will also present a result on a model with point interactions in three dimensions (arXiv: 1804.08295), which is too singular for classical renormalisation techniques to work.

Infinite dimensional integration techniques and applications to quantum mechanics

Sonia MAZZUCCHI
(Università degli Studi di Trento)

Techniques of infinite dimensional integration are widely used in several areas of theoretical physics, sometimes at a heuristic level. One of the main examples is provided by Feynman path integrals, which give an alternative description of quantum dynamics and allow the development of an heuristic calculus that works even when rigorous arguments fail. Despite the common belief that these techniques remain an heuristic tool, a rigorous mathematical definition can be achieved by implementing integration techniques on infinite dimensional spaces which go beyond measure theory and Lebesgue's "traditional" integration theory. In this talk I shall present the main mathematical results and give an overview of the several possible applications to quantum theories.

Why does (relativistic) quantum theory needs complex Hilbert spaces?

Valter MORETTI
(Università degli Studi di Trento)

I will present some recent results about the necessity of complex Hilbert space structures, in place of real or quaternionc ones, to formalize quantum theories in Hilbert space, starting from Solèr's theorem and Gleason's theorem, corrected to include quaternionic formulations and exploiting recent results of quaternionic spectral theory. In particular, I will discuss how Poincaré symmetry plays a crucial role to fix the Hilbert space complex structure when dealing with elementary systems.

A variational approach to epitaxially strained elastic films.

Massimiliano MORINI
(Università degli Studi di Parma)

We consider a variational model introduced in the physical literature to describe the epitaxial growth of an elastic film over a flat substrate when a lattice mismatch between the two materials is present. We first study quantitative and qualitative properties of equilibrium configurations, that is, of local and global minimizers of the free-energy functional. Next, we address the corresponding evolutionary problem, that is, the geometric evolution of the film profile by surface diffusion. We establish local-in-time existence and uniqueness of the flow for general (sufficiently regular) initial data. Finally, for a special class of initial data we prove the existence of a global-in-time solution and study the corresponding long-time behavior.

The Hardy inequality and the Dirac-Coulomb operator

Fabio PIZZICHILLO
(Basque Center for Applied Mathematics)

The free Dirac operator in $\mathbb{R}^3$ is defined as

$$H_0 = -i\alpha \cdot \nabla + m\beta \quad (\text{for } m > 0),$$

where $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ and β denote the so-called Dirac matrices.

This talk aims to show the connection between Hardy-type inequalities and the Coulomb-Dirac operator.

Firstly, I will prove some sharp Hardy-type inequalities related to the Dirac operator. Then I will show how to define the *distinguished* self-adjoint realisation of $H_0 + \mathbf{V}$, being $\mathbf{V}$ a Hermitian matrix potential with Coulomb decay.

Finally, I will focus on the spectral properties of Dirac-Coulomb operator. I will characterise its eigenvalues using the Birman–Schwinger principle. Moreover, I will show a bound from below of its discrete spectrum, and I will prove that this bound is reached if and only if $\mathbf{V}$ verifies some *rigidity* conditions.

This is a joint work with **B. Cassano**, and **L. Vega**.

Controlling the strain relaxation mechanisms in epitaxial thin films heterostructures

Alessia SAMBRI

(Università degli Studi di Napoli Federico II)

Perovskite oxides, described by the general formula ABO_3 , are recognised as one of the most fascinating class of compounds nowadays under investigation in theoretical and experimental solid state physics. This is due to three main reasons: their structural compatibility for the growth of epitaxial heterostructures, the variety of different physical properties they exhibit, and the constant improvement of oxides thin films growth techniques (such as PLD: Pulsed Laser Deposition) over the last decades. These three aspects paves the way to the fabrication of oxides thin films heterostructures in which specific device functionalities can be engineered for applications in microelectronics, microsensing, and so on.

Despite the progresses, an exhaustive understanding of all the aspects involved in a thin film deposition process is not available so far. In-plane lattice mismatch between compounds of different layers, small deviations from the ideal stoichiometry and oxygen content, and the intrinsic non-equilibrium nature of all film growth processes (which is extreme in PLD) contribute to the complexity of the problem.

In this talk, I will present experimental results on different strain relaxation mechanisms in two nominally identical oxides epitaxial heterostructures, composed by a $LaAlO_3$ film grown on $SrTiO_3$ substrate, grown in different conditions. The pure theoretical prediction on the value of the critical thickness above which the film of $LaAlO_3$ starts to relax its elastic energy (due to the in-plane lattice mismatch with $SrTiO_3$) fails in both cases to describe the experimental results. We demonstrate that it is possible to control the strain relaxation mechanisms, in order to induce either a smooth relaxation process, giving rise to uniform films well above the theoretical critical thickness, or to an abrupt relaxation process, eventually giving rise to a network of vertical cuts in the $LaAlO_3/LaAlO_3$ system, which result in a network of regularly shaped and upward-bent $LaAlO_3/SrTiO_3$ tiles. The final curvature of each $LaAlO_3/SrTiO_3$ tile can be predicted by minimizing the overall elastic energy of the bilayer with the constrain that $LaAlO_3$ and $SrTiO_3$ share the same lattice parameter at the interface.

Spectral Monotonicity for Schrödinger Operators on Metric Graphs

Cristian SEIFERT

(Technische Universität Hamburg)

During the last two decades there has been a tremendous study of differential operators on metric graphs. Recently, the behaviour of the spectrum of Laplacians or Schrödinger operators on metric graphs under perturbations of the geometry and/or topology of the graph came into focus.

In this talk we want study two of these so-called surgery principles, namely adding an edge (or more generally a graph) to a given one, and secondly joining two vertices into one single vertex. In order to do this we first classify all coupling conditions which are reasonable for the graph transformations in question and then explain the different ways of behaviours of the spectrum of the corresponding operators.

The talk is based on joint work with Jonathan Rohleder.

NLS ground states on metric trees

Paolo TILLI
(Politecnico di Torino)

Some recent results are presented, obtained in collaboration with S. Dovetta and E. Serra, concerning the existence of ground states with prescribed mass, for the NLS energy functional on unbounded metric trees. The nonlinearity in the NLS energy is a power with an exponent $p \in (2, 6)$. We show the existence of a critical mass μ_p such that no ground state of mass μ exists when $\mu < \mu_p$, while a ground state always exists when $\mu > \mu_p$.

Junctions of weakly-coupled ultracold systems

Andrea TROMBETTONI
(Scuola Internazionale Superiore di Studi Avanzati)

After briefly reviewing the use of ultracold atoms for the implementation of quantum devices, I discuss an example of junctions made by strongly interacting systems weakly coupled between them. In particular I discuss properties of 1D Bose gases and then of junctions of Tonks-Girardeau gases. When three Tonks-Girardeau gases are coupled, one can exactly map their Hamiltonian by means of a suitable Jordan-Wigner transformation into the Hamiltonian of the multichannel Kondo model. I will also show recent results on the experimental realization of Y-geometries with holographic traps, and comment about recent progress in atomtronics.

Invariant and quasi-invariant Gaussian measures associated with dispersive PDEs

Nicola VISCIGLIA
(Università di Pisa)

We discuss joint work with F. Planchon and N. Tzvetkov on statistical properties of the flow associated with dispersive PDEs.